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- Equivalently, dominance test: $\forall x_0', x_0'' \in \Omega_{X_0}$ check

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Classification with Bayesian networks

- Bayesian networks: a graph $\mathcal{G}$ to depict conditional independencies in $(X_0, \mathbf{X})$
- $\mathcal{G}$ induces a factorization in the joint

$$P(x_0, \mathbf{x}) = \prod_{i=0}^{n} P(x_i|\pi_i)$$

- Dominance test rewrites as

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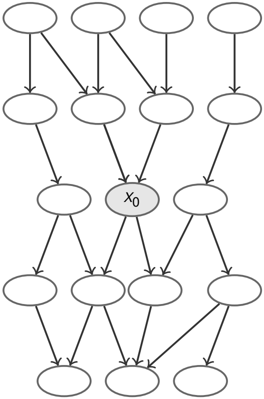
- Factors not including X_0 are equal to one focusing on the Markov blanket of X_0



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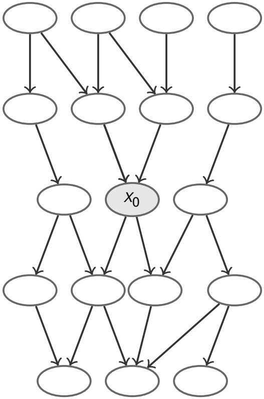

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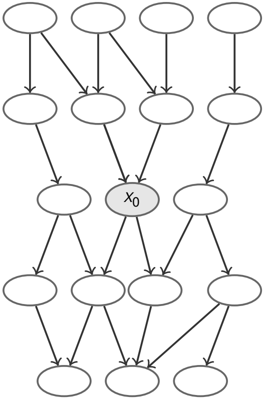

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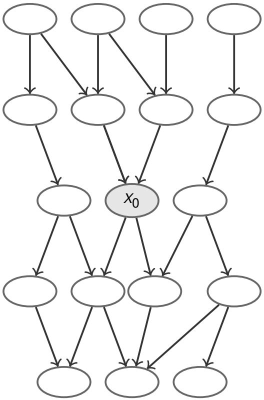

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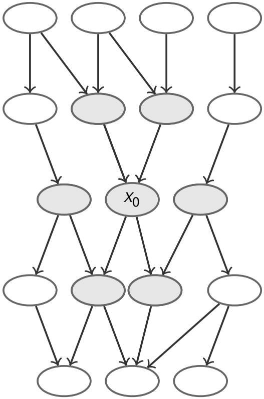

Classification with credal networks

- Few data $\Rightarrow$ unreliable learning of $P(X_i|\pi_i)$
- More reliable with sets of prob functions
- A credal set of joint $\mathbf{P}(X_0, \mathbf{X})$, instead of a single $P(X_0, \mathbf{X})$ (all Bayesian nets over $\mathcal{G}$)
- How to classify instances?
- x_0' dominates x_0'' iff this happens for each Bayesian net (maximality), i.e.,

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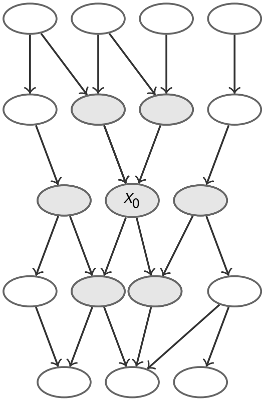

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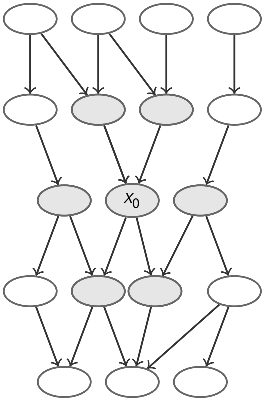

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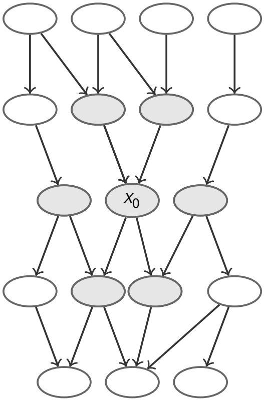

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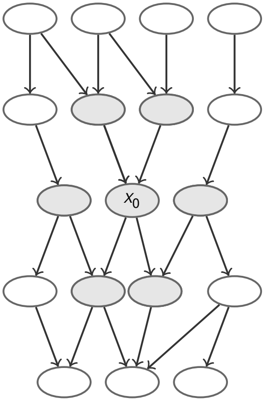

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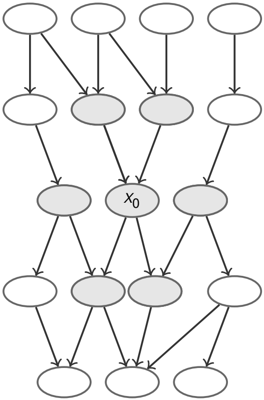

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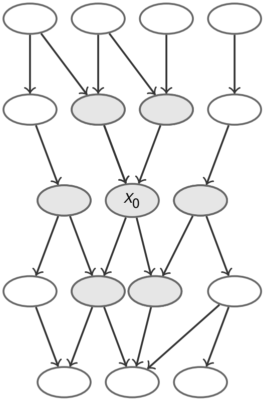

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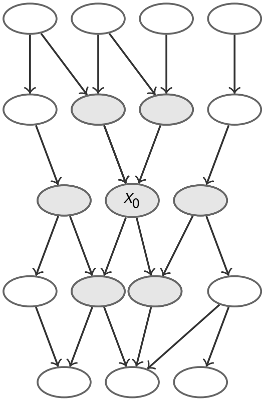
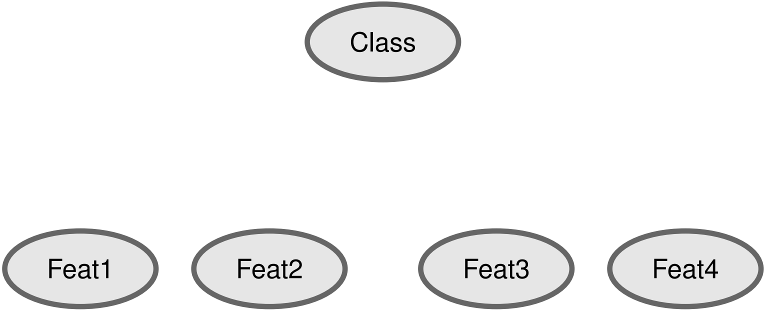
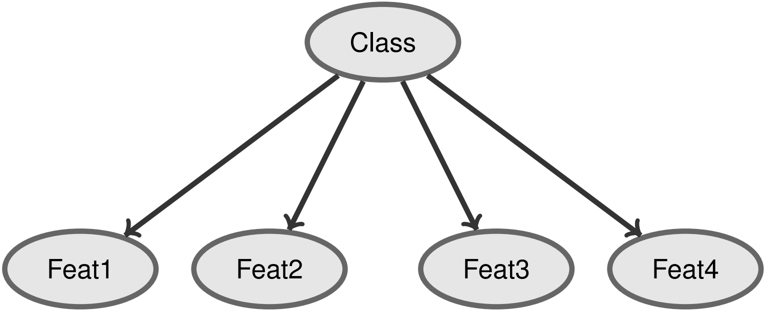
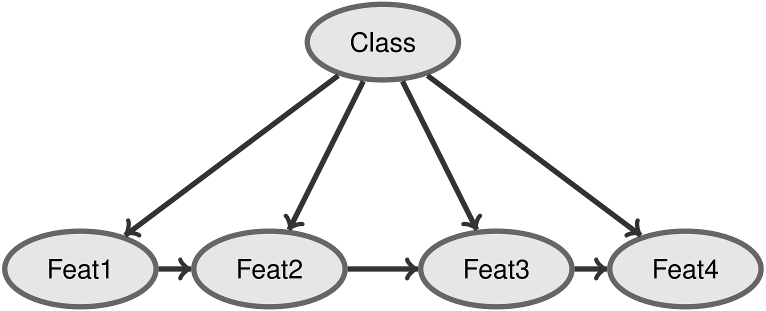



Class
Feat1
Feat2
Feat3
Feat4

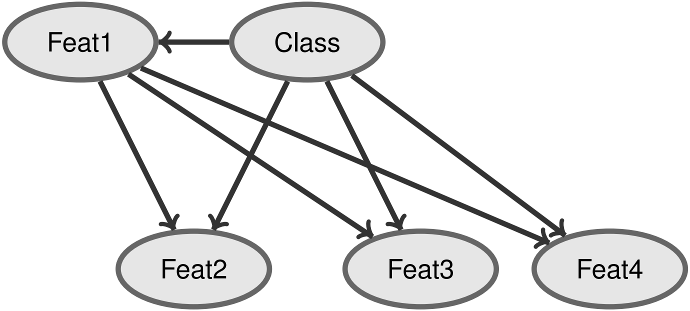
NAIVE



TREE-AUGMENTED NAIVE



ONE-DEPENDENCE (ODE)



Using IDM

Simplest case: naive with (local) IDM

$$\frac{P(c', f_1, f_2)}{P(c'', f_1, f_2)} = \frac{P(c')P(f_1|c')P(f_2|c')}{P(c'')P(f_1|c')P(f_2|c'')} > 1$$

- IDM: $P(c) = \frac{n(c)+st(c)}{N+s}$ with $0 \leq t(c) \leq 1$, $\sum_c t(c) = 1$

$$\min_t \frac{[n + st(c')] \cdot \ldots}{[n(c'') + st(c'')] \cdot \ldots} > 1$$

- Efficient optimization
  NAIVE CREDAL CLASSIFIER (Zaffalon & Corani)

  `http://www.idsia.ch/ giorgio/jncc2.html`

Metrics for Credal Classifier Perfomances

- Accuracy (% of correct classification) is not the only descriptor
- Single-Accuracy (accuracy if a single class is returned)
- Set-Accuracy (accuracy if multiple classes are returned)
- Determinacy (% number of instances with a single class)
- Average output size (average number of classes)
- Bayes-I (accuracy of the Bayesian if credal indeterminate)
- Bayes-D = Single-Accuracy
- Zaffalon et al. proposed a new utility-based measure (ISIPTA ’09)